

KS1 Maths Parent Workshop

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Aims of the today's session:

- Look at our calculation policy with a focus on the four operations (addition, subtraction, multiplication and division)
- Discuss how mathematics is taught through a CPA approach (Concrete - Pictorial - Abstract)
- Look at the concrete resources that we use at school to support mathematical teaching and learning
- Discuss the importance of oracy in maths and mathematical language
- An insight into the 'teaching for mastery' approach to mathematics
- How to support children in adopting a growth mindset in maths so they can achieve their potential.
- How to support your children at home with their maths learning

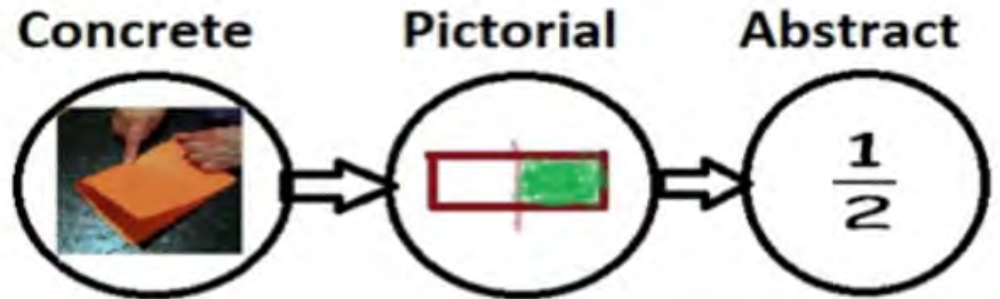
CPA Approach: Concrete Pictorial Abstract

- **Concrete:** 'doing' the maths - introducing real objects that can be manipulated to bring the problem to life. Eg: money, counters.
- **Pictorial** : 'seeing the maths' - making connections between the concrete and the pictorial representations and the pictorial and the abstract. Eg: part whole models, bar models, ten frames.
- **Abstract:** the ultimate goal is for children to understand abstract mathematical concepts, signs and notation. When a child demonstrates with concrete models and pictorial representations that they have grasped a concept, we can be confident that they are ready to explore the abstract.

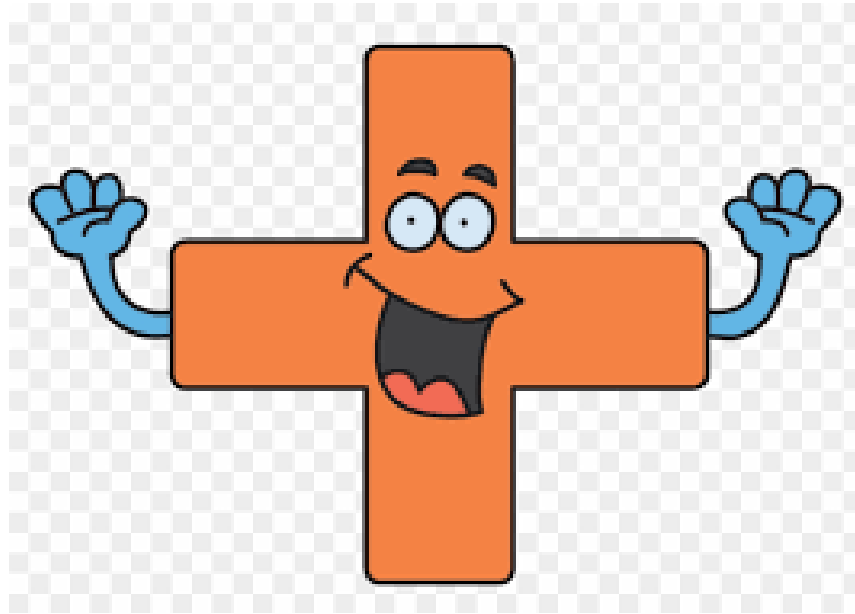


The CPA Approach

Maths should be practical for all ages and the CPA approach used at any time and with any age to support understanding



Addition in KS1



Calculation Policy

Year 1 - Addition

Concrete

- Use cubes (or practical equipment eg car park example from YR)



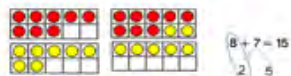
- Feely bag addition with Numicon:



- Numicon Balance



- Use of counters and 10-frames



- Use of Concrete bar model concrete using real objects or post it notes



Discrete – use 7 strips of paper

Continuous – two different lengths of paper eg 3 + 4

- Worded problems using concrete objects
"There are 8 cars. 2 more come long."

Pictorial

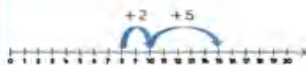
- Pictorial representation of Part-Part-Whole



- Bar Models (Discrete and Continuous)



- Counting on through use of number lines



Abstract

- Use of addition and equals signs to create simple number sentences

$$8 + 7 = 15$$

Oracy Sentence Stems:

_____ add _____ is equal to _____

The whole is _____. The parts are _____ and _____

To find the unknown part/whole I need to _____

- To count objects, children will use real objects.
- Numbers are represented through numicon.
- Children will use number lines to count on.

Concrete

- Continued use of Numicon and 10s frames



- Part Part Whole models using Base 10



- Addition of 10s using Base 10 *e.g.* $43 + 10$



- Moving to exchanging across tens and hundred *e.g.* $93 + 10$



- Base 10 and Cuisenaire Number Tracks for adding 10s *e.g.* $13 + 10$ and $23 + 20$



- Concrete bar models using Numicon and Base 10



Pictorial

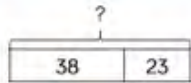
- Pictorial representations of Base 10



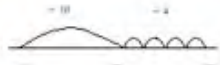
- Part Part Whole Models



- Bar Models (Continuous)



- Partitioning into 10s and 1s using number lines



Abstract

- Use of addition and equals signs to write number sentences of bonds up to 20 and derive fact families

$$\begin{array}{l} \text{e.g. } 13 + 7 = 20 \\ 7 + 13 = 20 \end{array}$$

Link to subtraction:

$$\begin{array}{l} 20 - 7 = 13 \\ 20 - 13 = 7 \end{array}$$

- Recognise Patterns in Addition Number Sentences

$$\begin{array}{l} 53 + 10 = 63 \\ 63 + 10 = 73 \\ 73 + 10 = 83 \\ 83 + 10 = 93 \\ 93 + 10 = 103 \end{array}$$

Oracy Sentence Stems:

The picture tells me I need to add the numbers.

The parts are known/unknown.

The whole is known/unknown.

I can partition ___ into ___ and ___.

___ ones/tens add ___ ones/tens = equal to ___.

I will exchange one ten for ten ones.

___ add ___ is equal to ___.

- Number bonds to 20
- Related number facts to 100
- Adding using concrete resources
- Commutative nature of addition (done in any order)
- Counting on using a number line

Pictorial representations using base 10

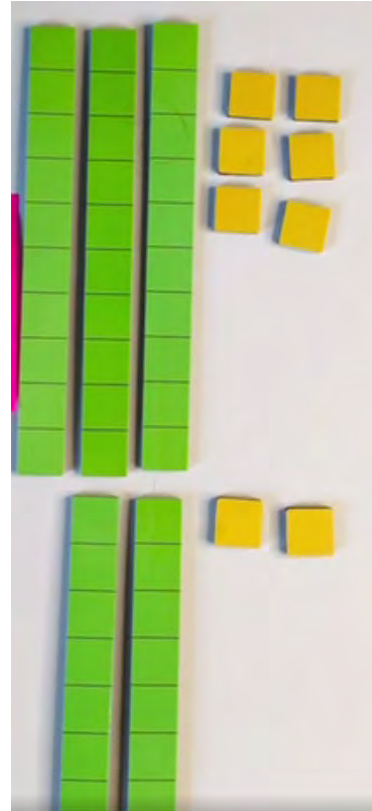
$$36 + 22$$

Can the children read the number

36 and represent it?

Can the children partition the number

into tens and ones?



The same calculation representation using place value counters

$$36 + 22$$

Can the children read the number

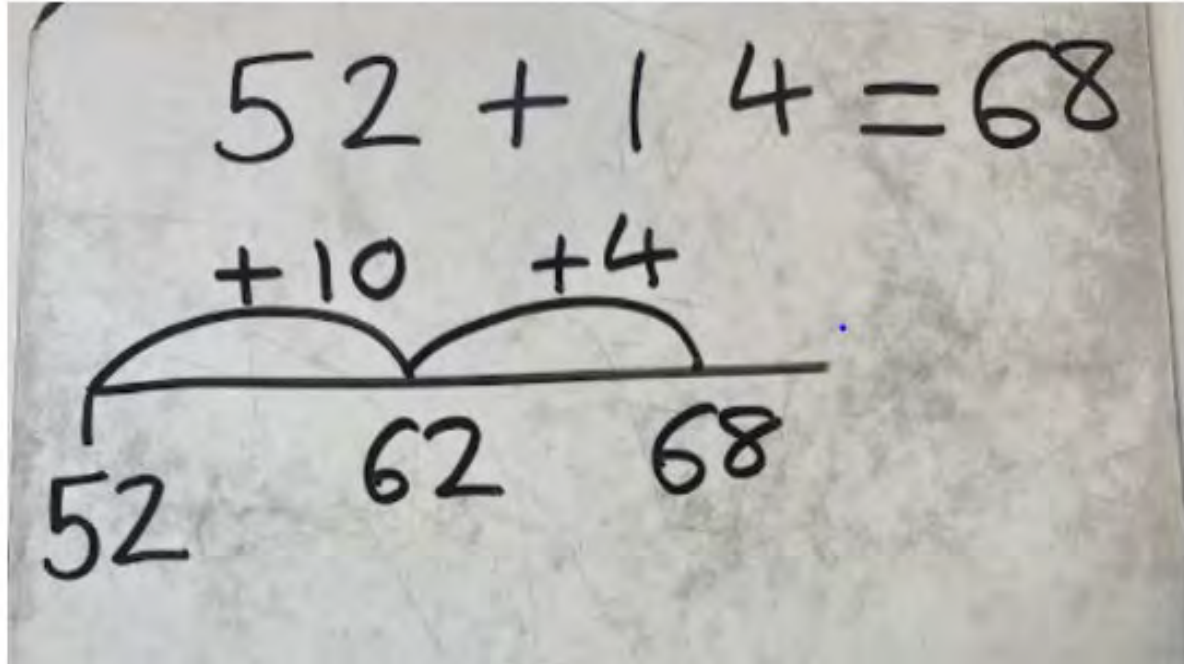
36 and represent it?

Can the children partition the number

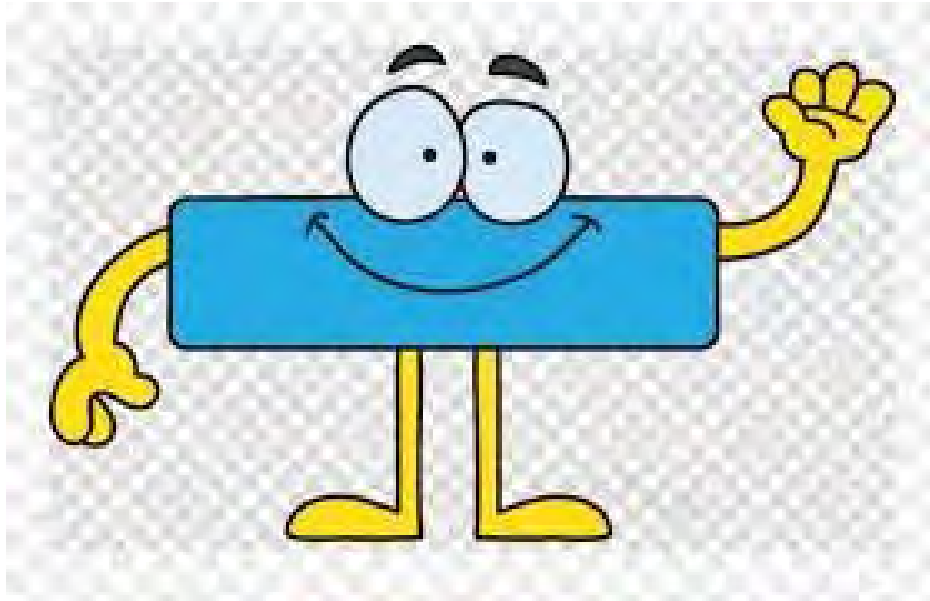
into tens and ones?



Number line addition - counting on



Subtraction in KS1



Concrete

- Squash Numicon into playdough and cut parts off – what's left? (see EYFS subtraction)

- Numicon and objects
eg, 6 – 2



eg, 9 – 3 = 6



- 5 frames
- Progressing to 10 frames



(Both Five-wise and Pair-wise)

- Chalk out number lines to count back on physically



- Number tracks

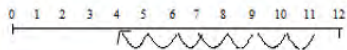


- Concrete Bar Models using real life objects (Comparison)

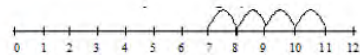


Pictorial

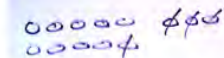
- Drawing jumps on prepared number lines eg, 11 – 7



- Find difference by drawing jumps on number lines



- Pictorial representations of objects crossed out



- Drawn Bar modelling from concrete objects (discrete)



- Provide children with squares with pictures of objects onto glue and stick discrete bar models and then cross out

Abstract

- Use of fingers
eg, Put 13 in your head, count back 4. What number are you at?
- Number sentences using – and =
- Subtract multiples of 10 eg, 50 – 20 = 30
- Relationships/Related facts (Summer Term)

Relationships/ Related Facts

$$\begin{array}{l} 5 - 2 = \square \\ 5 - \square = 3 \\ \square - 2 = 3 \\ \square - \square = 3 \end{array} \qquad \begin{array}{l} \square = 5 - 2 \\ 3 = \square - 2 \\ 3 = 5 - \square \\ 3 = \square - \square \end{array}$$

Oracy Sentence Stems:

_____ subtract _____ is equal to _____

When we subtract, we start with the whole

The whole is _____, The parts are _____ and _____

To find the unknown part/whole I need to _____

The difference between _____ and _____ is _____

1:1 correspondence


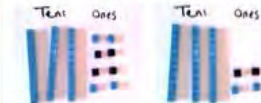


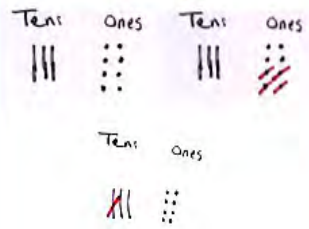



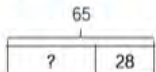
Linking objects to the value of their number

Finding the difference within 20

Number bonds to 20

Counting back on a 100 square or number line

Year 2 - Subtraction

| Concrete | Pictorial | Abstract |
|--|---|--|
| <ul style="list-style-type: none"> Numicon (Subtracting 10s)  Base 10 (Subtracting 10s and subtracting 1s)  Base 10 or place value counters with "Takesaway Pot"  Introduce the concept of exchange through moving objects physically  | <ul style="list-style-type: none"> Pictorial representation of Base 10  Counting back by partitioning when numbers aren't close together $46 - 18$  Find the difference (subtraction) by counting on  $42 - 27$ (add multiples of 10 first)  Continuous Bar models  | <ul style="list-style-type: none"> Bonds to 20 $\begin{array}{l} 13 + 7 = 20 \\ 7 + 13 = 20 \\ 20 - 7 = 13 \\ 20 - 13 = 7 \end{array}$ Derive facts families $\begin{array}{l} 13 + 7 = 20 \\ 7 + 13 = 20 \\ 20 - 7 = 13 \\ 20 - 13 = 7 \end{array}$ <div style="border: 1px solid #0070c0; border-radius: 15px; padding: 10px; background-color: #fff9c4;"> <p>Oracy Sentence Stems:</p> <p>The picture tells me I need to subtract the numbers.</p> <p>The parts are known/unknown:</p> <p>The whole is known/unknown:</p> <p>I can partition ___ into ___ and ___.</p> <p>___ ones/tens add ___ ones/tens is equal to ___.</p> <p>I will exchange one ten for ten ones.</p> <p>___ subtract ___ is equal to ___.</p> <p>When we subtract, we start with the whole</p> <p>___ and ___ have a difference of ___.</p> <p>___ and ___ have a difference of ___.</p> </div> |

- Partitioning the second number

- Count back the ones.

- Count back the tens.

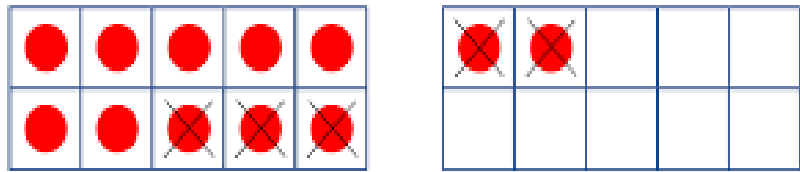
- Find the difference by counting up.

- Recognise the inverse

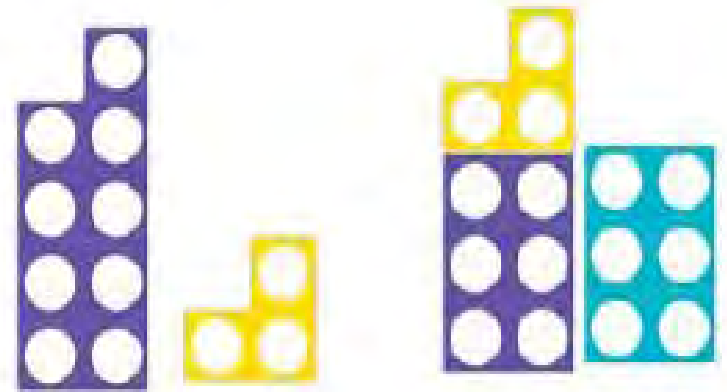
relationship between + and -

Subtraction with concrete resources

Using Ten Frames



For Subtraction



$$9 - 3 = 6$$

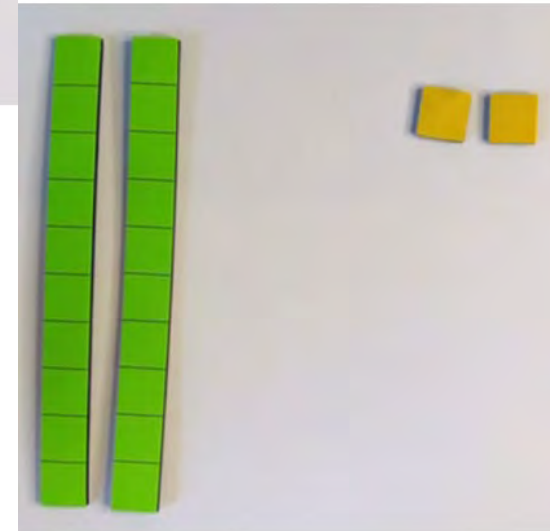
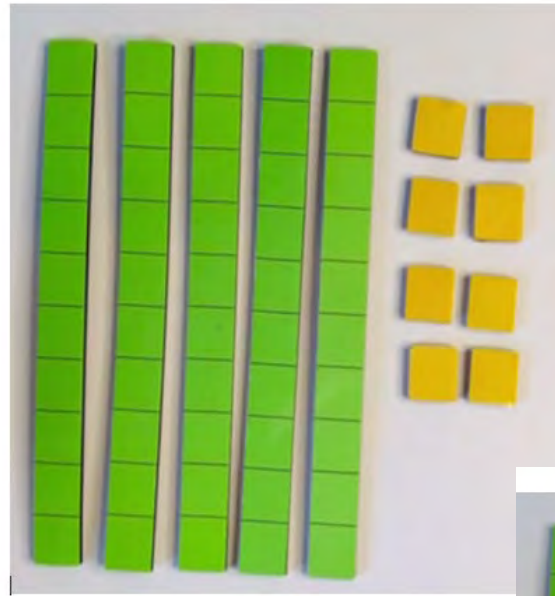
No exchange

$$58 - 36$$

In subtraction we only represent the first number.

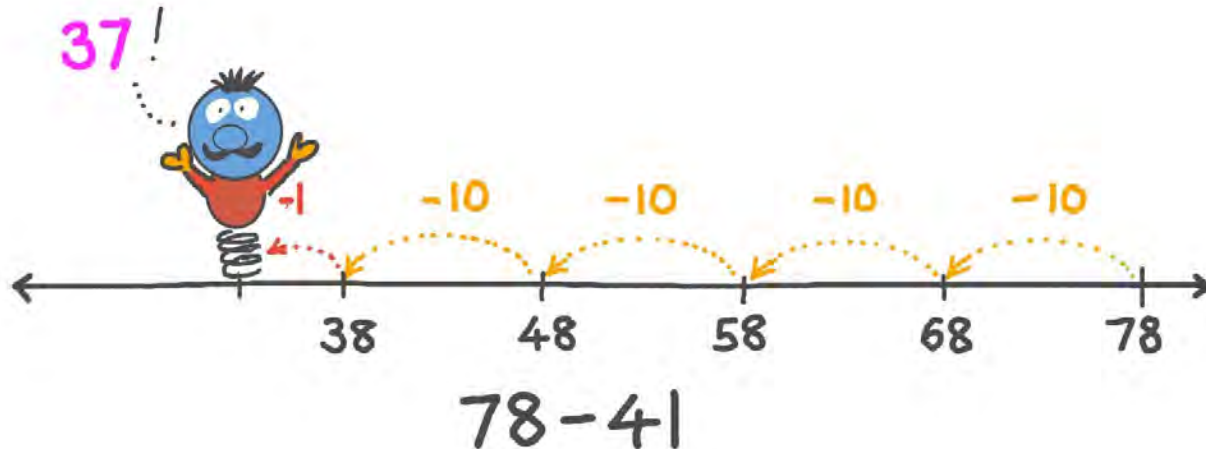
At the end of my calculation I will have less than I started.

Always take away the ones first!

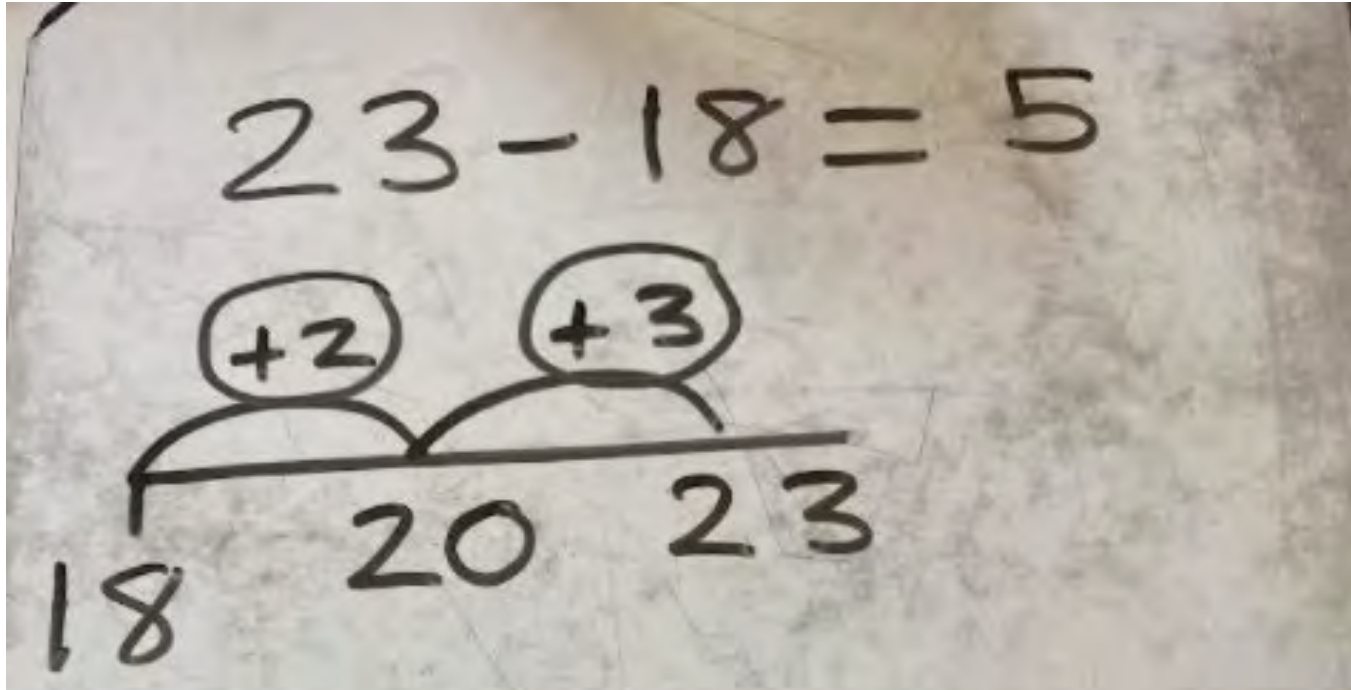


Counting back using partitioning

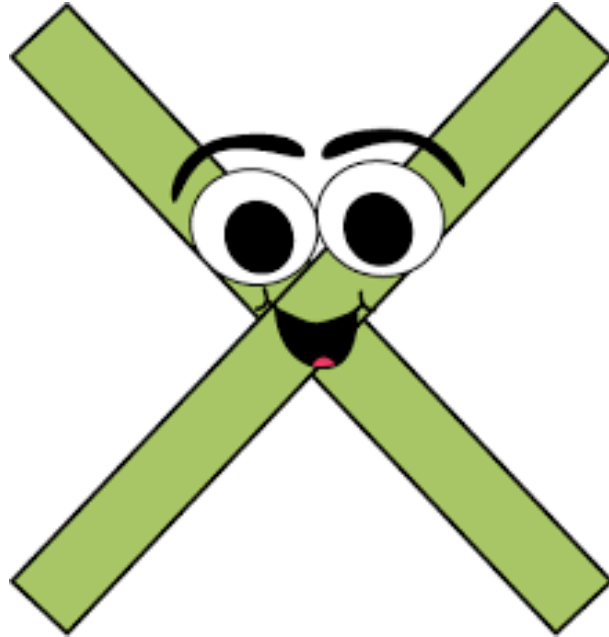
SUBTRACTING TWO-DIGIT NUMBERS
ON NUMBER LINES



Finding the difference using a number line - counting up


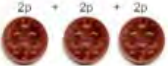

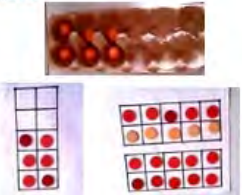



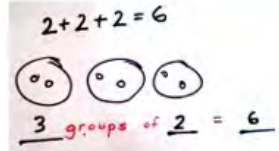


Multiplication in KS1



Year 1 – Multiplication

2's, 5's and 10's;
spotting the pattern using
manipulatives such as
numicon, number lines and
coins.

| Concrete | Pictorial | Abstract |
|--|---|--|
| <ul style="list-style-type: none"> Use different objects to add equal groups  $3 + 3 + 3$ Counting in multiples of 2p, 5p and 10p  $2p + 2p + 2p$ Numicon Doubles up to 10 + 10  Use of arrays and 10s frame to count in multiples of 2s, 5s and 10s  Jumping on chalk number lines  | <ul style="list-style-type: none"> Pictorial representation of groups of concrete objects <p>There are 2 sweets in one bag. How many sweets are there in 5 bags?</p>  Use of number lines  | <ul style="list-style-type: none"> Repeated addition sentences of concrete objects and pictures  <p>Oracy Sentence Stems:</p> <p>___ groups of ___ are equal to ___</p> <p>The pattern is increasing in ___</p> <p>There are ___ groups of ten. There are ___ ones.</p> <p>___ groups of ten are equal to ___</p> <p>___ groups of two are equal to ___</p> <p>___ groups of five are equal to ___</p> |

Very important that this maths knowledge builds through concrete meaningful contexts using concrete objects.

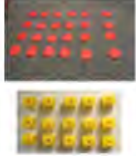
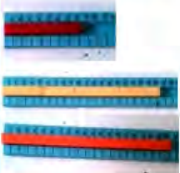
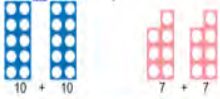

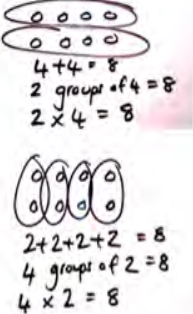
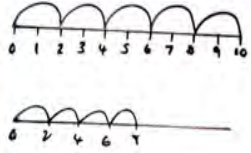
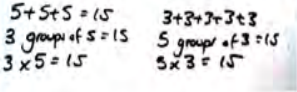
Repeated addition.

Knowing the multiplication facts for the 2, 5 and 10 times tables.

Using the X symbol

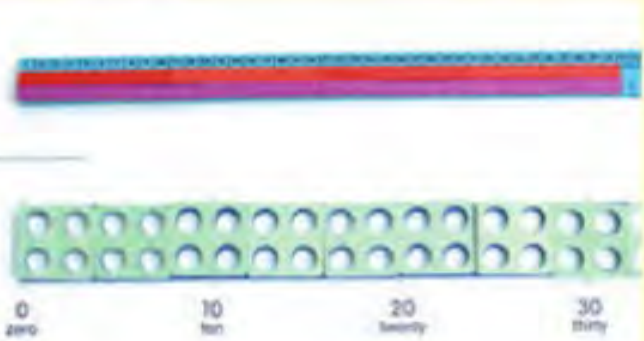
Recognise that multiplication is commutative (can be done in any order)

Year 2 – Multiplication

| Concrete | Pictorial | Abstract |
|---|--|---|
| <ul style="list-style-type: none"> Use of arrays to show multiplication sentences  Cuisenaire tracks (2s, 5s, 10s)  Doubles of all numbers up to 20 by partitioning and recombining ex. 17 + 17  Jumping on Chalk Number Lines  | <ul style="list-style-type: none"> Drawn arrays in different rotations (to show commutativity)  Use of number lines (discrete moving to continuous)  | <ul style="list-style-type: none"> Number sentences of repeated addition moving to use of x symbol  <div style="border: 1px solid #4a86e8; border-radius: 15px; padding: 10px; background-color: #fce4d6; margin-top: 10px;"> <p>Oracy Sentence Stems:</p> <p>There are ___ parts with a value of ___</p> <p>The whole is: ___</p> <p>___ groups of ___ is equal to ___</p> <p>___ multiplied by ___ is equal to ___</p> </div> |

Understanding multiplication

Counting in groups of...



Spotting patterns

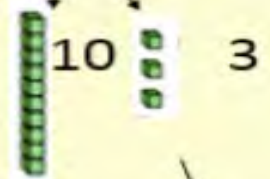
| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Doubling (and halving!)

$$3 + 3 = 3 \times 2$$



$$13 \times 2 =$$

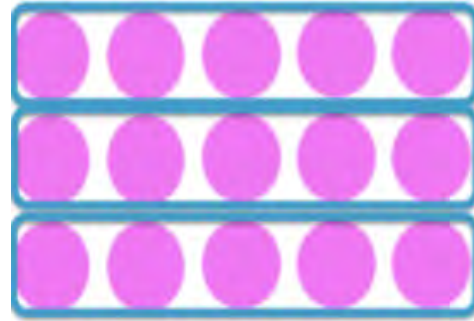


20

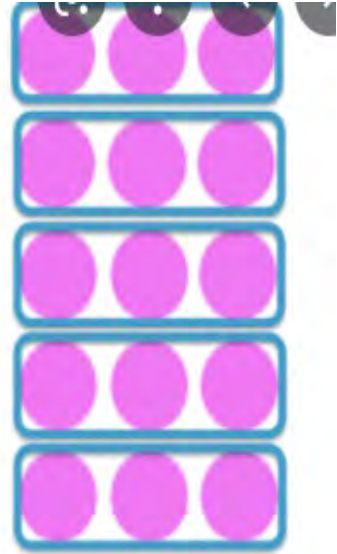


6

Arrays



$$3 \times 5 = 15$$



$$5 \times 3 = 15$$

Repeated addition



$$2 + 2 + 2 = \square \quad 3 \times 2 = \square$$



$$4 + 4 = \square \quad 2 \times 4 = \square$$



$$3 + 3 = \square \quad 2 \times 3 = \square$$

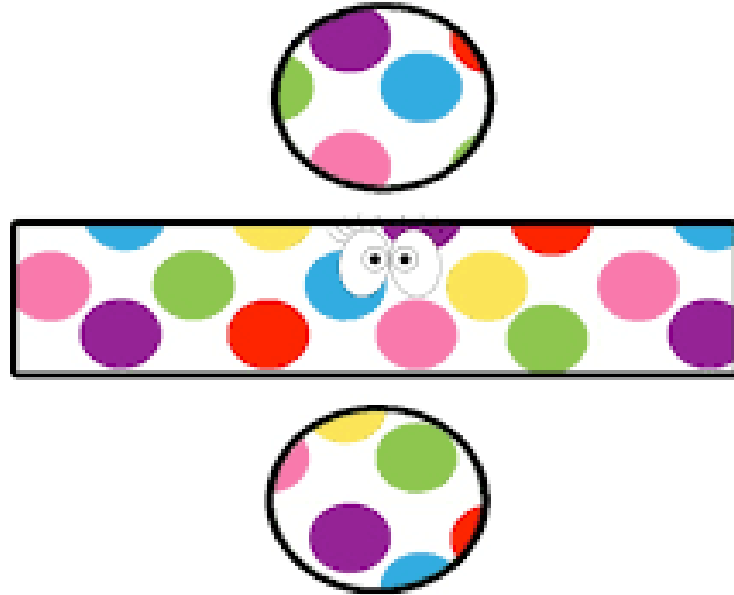


$$3 + 3 + 3 = \square \quad 3 \times 3 = \square$$



$$1 + 1 + 1 + 1 = \square \quad 4 \times 1 = \square$$

Division in KS1





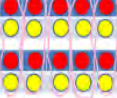




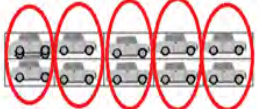
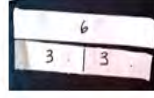


Year 1 – Division

Division as sharing through practical activities.

Halving even numbers up to 10

Not introduced to the \div symbol until year 2











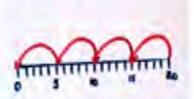
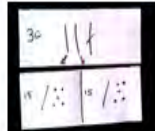
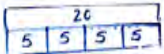
| Concrete | Pictorial | Abstract | Guidance |
|--|---|--|---|
| <ul style="list-style-type: none"> Sharing/halving even numbers between 2 up to 10 and 20 using concrete objects  Sharing equally using concrete objects <i>eg</i> There are 3 bowls and 9 bananas, how many bananas are there in each bowl?  Sharing concrete objects using concrete bar models  Moving to grouping <i>eg</i> How many groups of 2 in 6? <i>eg</i> How many groups of 5s in 10?  Use of 10 frames to group <i>eg</i> How many groups of 4 in 20?  | <ul style="list-style-type: none"> Use of pictures or shapes to share quantities    Use of bar models to share equally  Use of 10 frames and pictures in 10 frames to group  | <ul style="list-style-type: none"> Understanding $\div 2$ as half Understanding division as the inverse of multiplication Divide 10 in 5 groups. How many in each group? Abstract bar models  | <p>As with EYFS, division should be taught first through the concept of sharing amounts equally and then progressing to grouping objects.</p> <p>Make use of sharing and groups in illustrations <i>eg</i> Beans in Jack and the Bean Stalk, Food in Teddy Bear's Picnic.</p> <p>See NC objectives for fractions.</p> <div data-bbox="1400 758 1680 1010" style="border: 1px solid orange; border-radius: 15px; padding: 10px; margin-top: 20px;"> <p>Oracy Sentence Stems:</p> <p>_____ shared into _____ equal parts. _____ is _____.</p> <p>_____ shared equally into groups of _____ makes _____ groups.</p> <p>I shared _____ into _____ equal groups. There are _____ in each group.</p> <p>The pattern is increasing in _____.</p> <p>The pattern is decreasing in _____.</p> <p>There are _____ groups of _____. There are _____ ones.</p> <p>There will be _____ in each group.</p> </div> |

Recalling division facts for 2,5 and 10 times tables

Division as sharing and grouping.


Understanding \div 2 as 'half of'

Recognise relationship between X and

| Concrete | Pictorial | Abstract | Guidance |
|---|--|--|---|
| <ul style="list-style-type: none"> Share out quantities into equal groups using cubes, counters and other objects.  Use of Numicon and Base 10 for halving numbers (sharing)  Use of Numicon for grouping. How many groups of 5 in 20?  Use of contextual questions: 20 children going on a school trip. Each car holds 5 people; how many cars are needed?  Use of arrays:  There are 20 <u>cabbages</u> and they are put into rows of 5. How many rows are there?  Concrete Bar Models to represent division  Use of Cuisenaire number tracks eg, $15 \div 5$  | <ul style="list-style-type: none"> Grouping  Use number tracks/lines for questions eg, How many 2s in 10?  <p>"How many 5s are there in 20?"</p>  Pictorial representations of bar models  Bar models (continuous)  | <ul style="list-style-type: none"> Understand '÷ 2' as 'half of'. Understand that '÷ 4' as 'quarter of'. Understand that division is not commutative. Recognise relationship between \times and \div Record using division (\div) and equals (=) signs. Divide 10 into 5 groups. How many are in each group? <div data-bbox="1506 666 1845 971" style="border: 1px solid black; border-radius: 15px; padding: 10px; margin-top: 20px;"> <p>Oracy Sentence Stems:</p> <p>_____ shared into _____ equal parts _____ is _____</p> <p>_____ divided by _____ is equal to _____.</p> <p>We exchange 1 ten for 10 ones.</p> <p>When we divide, the whole is known and the number or parts or the value of the parts is unknown</p> <p>_____ divided by _____ is equal to _____.</p> </div> | <p>Division should be taught first through the concept of sharing amounts equally and then progressing to grouping objects.</p> |

Division as sharing

$15 \div 3 = 5$

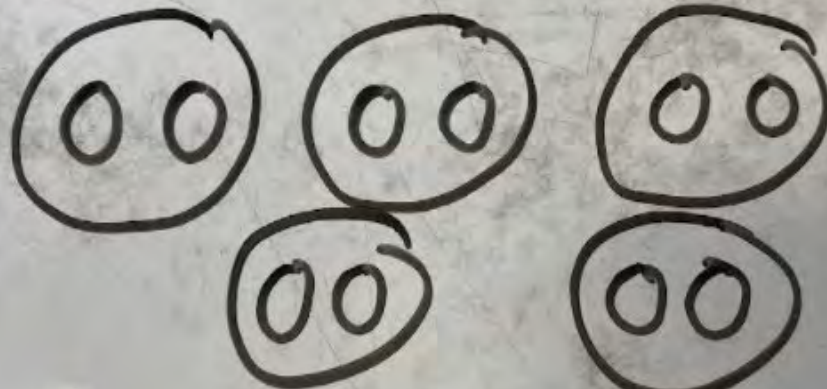


oooooooooooooooooooo

There are 15 altogether
There are 3 groups
There are 5 in each
group.

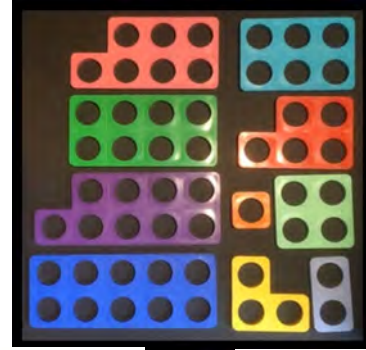
Division as grouping

How many groups
of 2 in 10?



Manipulatives - concrete resources

Numicon



Dienes

Place value counters

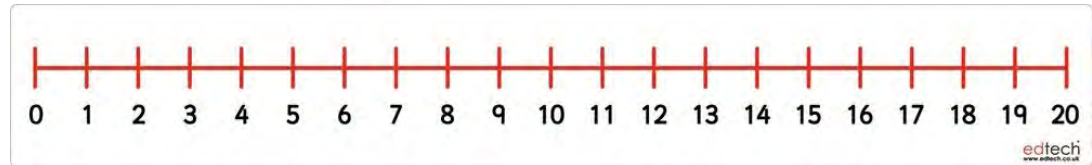
100 square

Number lines

Coins



| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

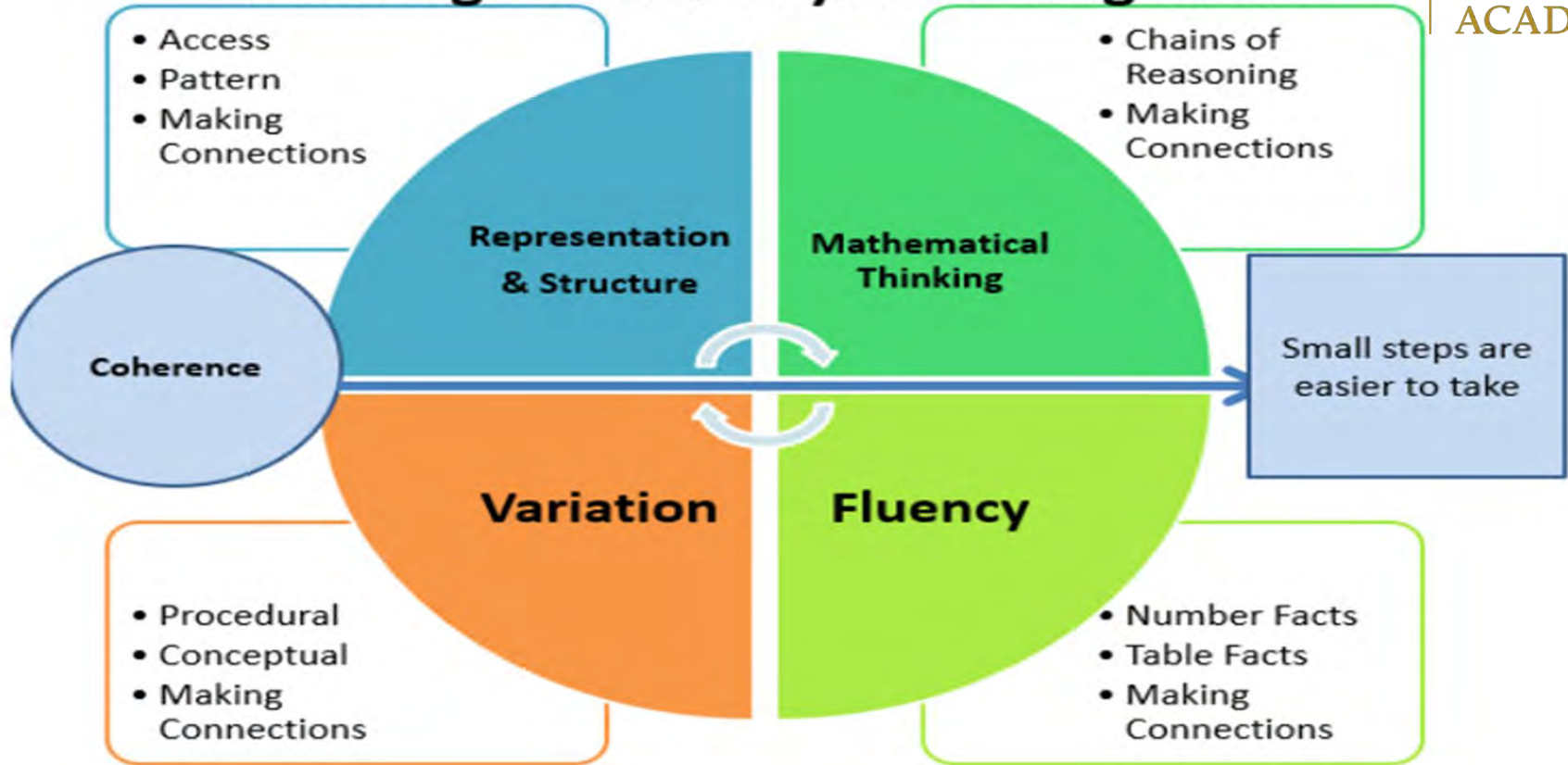


The Teaching for Mastery Approach

What does it mean to master something?

- I know how to do it
- It becomes automatic and I don't need to think about it
- I'm really good at it- painting a picture
- I can show someone else how to do it

Teaching for Mastery: The 5 Big Ideas



Making generalisations

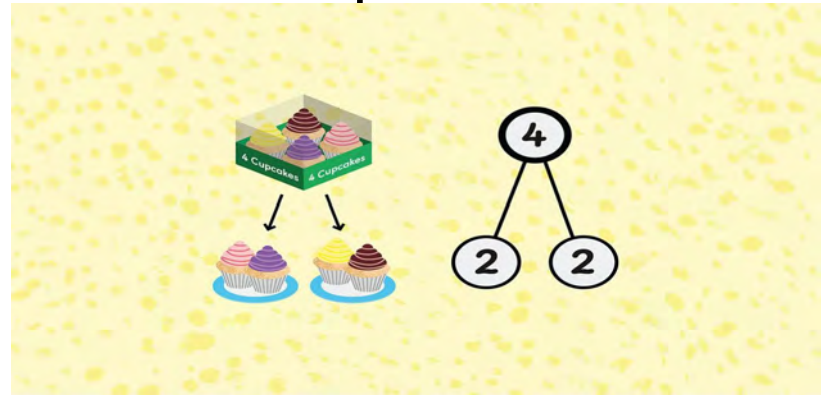
- If you change the position of the numbers in a multiplication calculation, the answer will always stay the same.

E.g. $4 \times 5 = 20$ and $5 \times 4 = 20$ (commutativity)

- All even numbers end in 0, 2, 4, 6, 8
- When counting in 10s, the ones digit always stays the same but tens digit changes

Representation and Structure

- Representations are used in lessons to expose the mathematical structure being taught.
- In essence representation refers to the wide variety of ways to capture an abstract concept or relationship.



Multiple representations of the same number.

| | | | |
|-------------|-------------|--------------------------------|--|
| Number | | Number word | |
| 47 | | Forty seven | |
| Draw it | | Expanded form | |
| Tens | Ones | $40 + 7 = 47$ $7 + 40 = 47$ | |
| | | | |

Part part whole models

27.09.16

by myself | with partner | with adult 1.

Hot

WALT use a part-whole model to partition 6

1 5 4 2 3 3 4 2

6 6 6 6 6 6 6 6

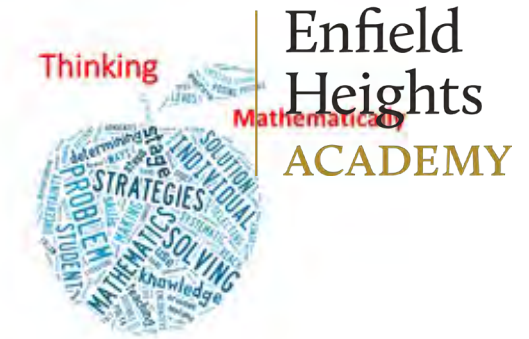
Hold on... look there are 7 ways for the number 6. My idea works again!

Leo's conjecture

If 6 is the whole.

What are the parts?

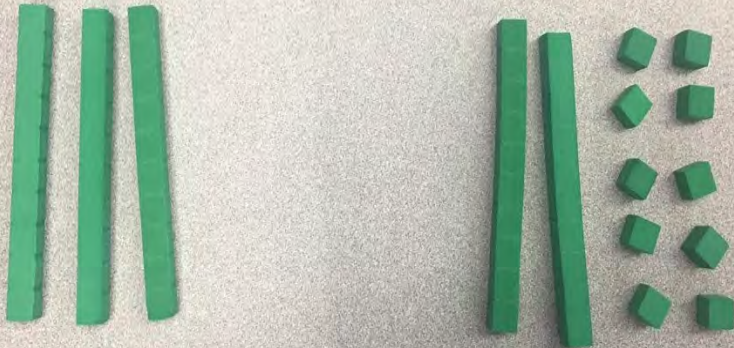
Mathematical Thinking



- If taught ideas are to be understood deeply, they must not merely be passively received but must be worked on by the pupil: thought about, reasoned with and discussed with others.
- We provide lots of opportunities for peer and collaborative discussions in our daily maths lessons.
- Problem solving and reasoning opportunities in every session to embed a depth of learning

Reasoning: What's the same and what's different?

What is the same? What is different?



What is the same?
What is different?

$$7 + 3 = 10$$

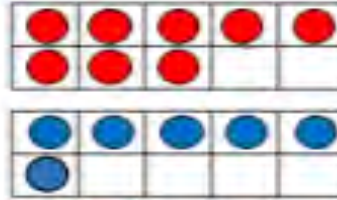
$$17 + 3 = 20$$

$$20 = 7 + 13$$

Explain your thinking.

Reasoning: Spotting mistakes and misconceptions

Dexter uses ten frames to calculate eight plus six.



He says,



$$8 + 6 = 16$$

Do you agree?
Explain why.

Reasoning: True or false

True or false?

12 is an odd number.

Prove your answer using concrete,
pictorial and abstract representations.
Explain each approach.

The logo for Times Tables Rockstars features the words 'TIMES TABLES' in blue and 'ROCKSTARS' in pink, both in a stylized, jagged font. Below the logo, it says 'BETA PREVIEW' and 'Beta site is currently for inspection by your partner. Any content you shared previously is hidden. @times-tables.com for more support links.' There is also a 'Log in now' button.

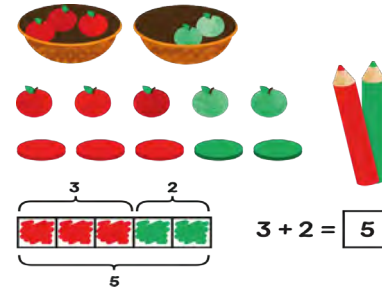
Fluency



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- Quick and efficient recall of facts and procedures and the flexibility to move between different contexts and representations of mathematics.
- Hit the button - Topmarks for quick fire number fact practice
- TT Rockstars- year 2
- Number bond work - noticing the patterns, for example:
 $7 + 3 = 10$ so $70 + 30 = 100$

Conceptual variation

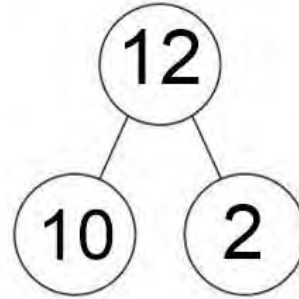


- Variation is all about how the teacher represents the concept being taught
- Provides opportunities to work on different representations of the same mathematical idea.
- For example, looking and multiple representations of the number 54 with dienes, place value counters, arrow cards, 100 square etc.

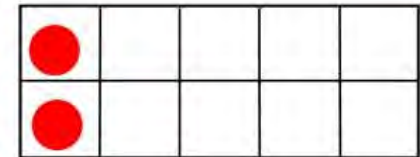
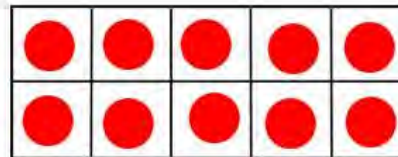
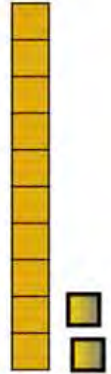
Variation helps visualisation

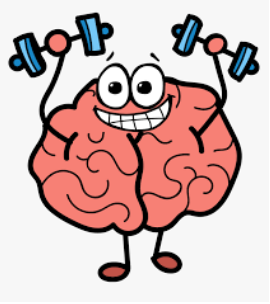
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12





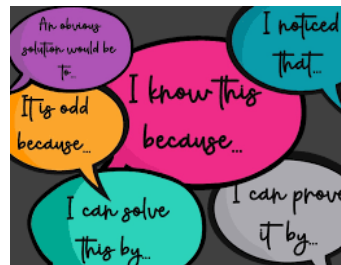
Everyone Can!



At Enfield Heights we encourage children to develop a **growth mindset** by using these strategies:

- It's ok to get it wrong- mistakes are valuable opportunities to re think and understand more deeply. Spotting and sharing mistakes between teachers and pupils makes learning richer.
- Praising hard work- is a great motivator by focusing on effort rather than success. Children will be more willing to try harder and take risks.
- Mind your language- the language we (teachers and parents/ carers) use around learners has a profound effect on their mindsets. Make a habit of using growth phrases like 'everyone can', 'mistakes can help you learn', 'just try for a little longer' and the key of them all- 'yet'. 'I just cannot solve this yet!'

Maths Talk



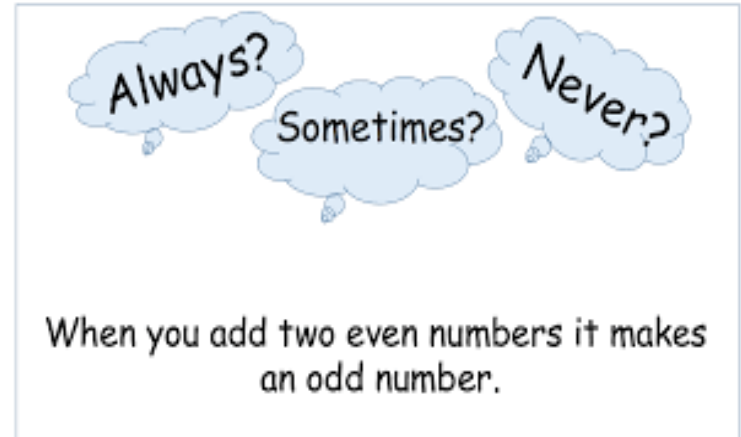
- **Key Vocabulary:** Discussing essential vocabulary
- **Full sentences** : Teachers and children need to use full sentences to explain or respond. When children use complete sentences, it both reveals their understanding and embeds their knowledge.
- **Stem sentences:** These help children express mathematical concepts accurately and scaffolds their responses.

Eg: *'4 is a part, 5 is a part, 9 is the whole.'*

- **Consistency:** all use same mathematical terms in full, i.e ones instead of units

Ways to encourage maths talk at home

- Why is that a good mistake?
- If we know this, what else do we know?
- Give me . . .tell me . . .show me . . .
- Why is this the odd one out?
- The answer is . . .what is the question?
- Give me a silly answer for . . .?
- Always, sometimes, never true?



Any questions?

